

## Robust Time-Delay Control of a Reclaimer

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(Received March 6, 1999)

In this paper, a robust time delay control for a reclaimer is investigated. Supplying the same amount of raw material throughout the reclamation process, from the raw yard to a sinter plant, is important to keep the quality of the molten steel uniform. As the parameter values of the reclaimer are not available, the boom rotational dynamics is modeled as a second order differential equation with unknown coefficients. The unknown parameters in the nominal model are estimated using recursive estimation method. Another important factor in the control problem of a reclaimer is the large time delay in output measurement. Assuming a multiplicative uncertainty, that accounts for both the unstructured uncertainty neglected in the modeling and the structured uncertainty in the parameter estimation, a robust Smith predictor is designed. A robust stability criterion for the multiplicative uncertainty is derived. Following the work of Goodwin *et al.* (1992), a quantifying procedure of the multiplicative uncertainty bound, through experiments, is described. Experimental and simulation results are provided.

**Key Words:** Reclaimer, Modeling, Identification, Smith Predictor, Robustness, Model Uncertainty

### 1. Introduction

The reclaimer is a piece of industrial equipment that excavates and transports raw materials, like coal and iron ore, in the raw yard of a steel plant. The reclaimer consists of a main body that translates on a rectilinear rail, a boom rotating vertically and horizontally on the main body, and a tilted rotating disk at the end of the boom. The boom is approximately 50 m in length, and the rotating disk is about 6 m in diameter. Scooping buckets are attached to the circumference of the circular disk. The disk tilts at various angles allowing the raw material in the buckets to fall on the conveyor belt. Which is located in the middle

section of the boom as the disk rotates (Hong *et al.*, 1997; Choi *et al.*, 1999).

Currently, all stockyard reclaimers are manually operated. When the operator knows the kind, quantity, and location of the raw material to be excavated, he manually drives the reclaimer to the given spot and scoops up the raw material. However, skilled operators are rare and in times of poor visibility or darkness, it is difficult for the operator to land the buckets on the desired spot of the ore pile. Also most stockyards are full of dust with unsuitable working conditions.

In this paper, as a part of reclaimer automation, a mathematical model of the reclaiming process and a robust Smith predictor design are investigated. A mathematical model of the reclaiming process including boom rotational dynamics is proposed. The control objective in this paper is to provide the sinter plant with a constant quantity of raw material throughout the reclamation process. As the parameter values of the reclaimer are not available, the boom rotational dynamics is modeled as a second order differential equation with unknown coefficients. The unknown coeffi-

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coefficients are estimated by a recursive estimation method. The dynamics neglected in the modeling, such as the spool valve dynamics and the hydraulic pressure dynamics, are estimated from experiments. By analyzing the shape of a raw material pile and the boom's circular motion, an output equation is derived as a function of the angular displacement and velocity of the boom. By combining the second order boom rotational equation with the estimated parameters and the output equation, a nominal plant model is defined.

Another important factor for reclaimer control is the large output measurement delay caused by the non-collocation of the end-effector (an excavating bucket) and the output sensor (a load cell). The load cell is located under the conveyor belt and is approximately 5 m from the rotational axis of the disk. The structure of a Smith predictor as a feedback controller is investigated. To design a robust Smith predictor which incorporates both the dynamics neglected in the model and the uncertainties in the parameter estimation, a multiple uncertainty is added to the nominal plant model. A robust stability criterion of the Smith predictor, with multiplicative uncertainty, is derived. By following the work of Goodwin *et al.* (1992), a systematic procedure of quantifying the uncertainty bound is illustrated. This procedure is then applied to the reclaimer. Finally, the closed loop performance of the robust Smith predictor is simulated.

Contributions of this paper are as follows. First, to the best of authors' knowledge this paper is the first investigation of reclaimer control to appear in the literature. Second, by estimating the uncertainty bound of the neglected dynamics, the robustness of the Smith predictor has been enhanced.

This paper has the following structure. In Sec. 2, the boom rotational dynamics is modeled and its parameters are identified. By analyzing the reclaiming process, an output equation is derived and a nominal plant model is defined. In Sec. 3, a robust Smith predictor is investigated and a robust stability criterion, with multiplicative uncertainty, is derived. In Sec. 4, a systematic procedure of estimating the uncertainty bound from

experiment is described. Experiments and simulations are provided in Sec. 5. Conclusions are given in Sec. 6.

## 2. Modeling

In this section a mathematical model of the reclaimer which will be used for the control system design is investigated. It must be noted that the values of the system parameters such as the mass of the boom, the servo valve flow gain, etc., are not available. By combining the parameters of the boom rotational mechanism and the parameters of the hydraulic drive system, the dynamic equation of the boom rotational motion is formulated as a second order differential equation with unknown coefficients. These unknown coefficients are determined using a recursive estimation method which employs experimental input and output data. A mathematical expression for the reclaiming rate, as a function of the angular displacement and the angular velocity of the boom, is also proposed. An output equation will be derived by analyzing the trajectories of the buckets and the geometry of the material pile. Finally, by combining the second order boom rotational equation with the estimated parameters and the output equation (after linearization), a nominal plant model for the control system design is proposed.

### 2.1 Boom rotational dynamics

Figure 1 shows a reclaimer and its schematic diagram depicting the boom rotational motion. The reclaimer is composed of a hydraulic power supply, a servo valve, a hydraulic motor, a ring gear and a boom structure. The input to the boom rotational system is a current,  $i$ , to the hydraulic servo valve and the output is the horizontal rotational speed,  $\omega_2$ , of the boom. If the motor cross-port leakage and the pressure drop in the pipes are neglected, the following continuity equations hold (Watton, 1989).

$$(k_o + k_{fi})\sqrt{p_s - p_1} = D_m\omega_2 + \frac{p_1}{R_e} + \mu\frac{dp_1}{dt} \quad (1)$$

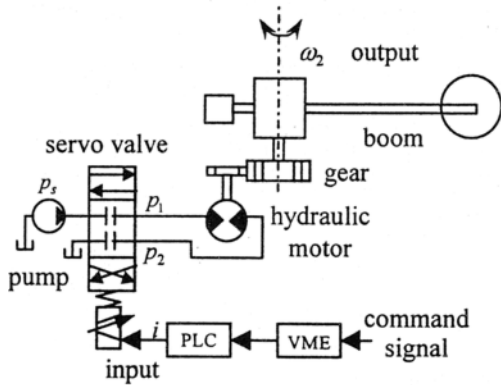


Fig. 1 A reclaimer and its schematic diagram depicting the boom rotational motion.

$$(k_o + k_f i) \sqrt{p_2} = D_m \omega_2 - \frac{p_2}{R_e} - \mu \frac{dp_2}{dt} \quad (2)$$

where  $k_o$  and  $k_f$  are the flow coefficients of the servo valve,  $p_s$  is the supply pressure,  $p_1$  and  $p_2$  are the pressures of the input and output ports of the hydraulic motor, respectively,  $D_m$  is the displacement per radian of the hydraulic motor,  $R_e$  is the motor resistance, and  $\mu$  is the flow capacitance.

According to Newton's second law, the equation for the boom slew motion is

$$D_m (p_1 - p_2) = T_L + B_v \omega_2 + J_b \frac{d\omega_2}{dt} \quad (3)$$

where  $T_L$  is the external load torque,  $B_v$  is the viscous friction coefficient, and  $J_b$  is the boom inertia. Assuming that the external load torque  $T_L$  is negligible, the transfer function from  $i$  to  $\omega_2$  of the linearized system at an operating point is

$$\frac{\delta\omega_2(s)}{\delta i(s)} = \frac{\Gamma \mu}{2} \frac{k_i / D_m}{s^2 + \left[ \frac{\Gamma}{2R_v} + \frac{\Gamma}{2R_e} + \frac{\mu R_f}{2} \right] s + 1 + \frac{R_f}{2R_v} + \frac{R_f}{2R_e}} \quad (4)$$

where  $k_i$  is the servo-valve flow gain in the steady state,  $\Gamma = J_b / D_m^2$  is the inductance,  $R_v$  is the servo-valve resistance in the steady state, and  $R_f = B_v / D_m^2$  is the viscous resistance. It is noted that all the values of  $k_i$ ,  $\Gamma$ ,  $\mu$ ,  $R_e$ ,  $R_v$ , and  $R_f$  in Eq. (4) are not known. This is because the particular reclaimers for which the control system has been designed are currently operating in the field. From Eq. (4), a second order nominal model of the boom rotating system, with unknown coefficients  $a_1$ ,  $a_0$ , and  $b_0$ , is defined as

$$\frac{\omega_2(s)}{i(s)} \triangleq G_o(s) = \frac{b_0}{s^2 + a_1 s + a_0} \quad (5)$$

The unknown coefficients in Eq. (5) are identified through experiment. Let the discrete transfer function of Eq. (5) be (Phillips and Nagle, 1995)

$$G_o(z) = \frac{\omega_2(k)}{i(k)} = \frac{b'_0 z^{-1}}{1 + a'_1 z^{-1} + a'_0 z^{-2}} \quad (6)$$

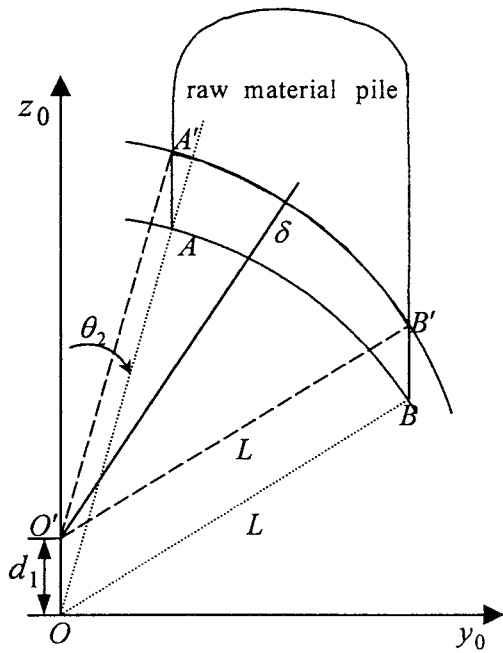
Let  $\theta \triangleq [a'_1 \ a'_0 \ b'_0]^T$ . Then the estimate,  $\hat{\theta}_N$ , of  $\theta$  with  $N$  experimental data can be obtained by an appropriate parameter estimation algorithm such as a recursive least squares estimation method that satisfies

$$\hat{\theta}_N = \underset{\theta}{\operatorname{argmin}} \frac{1}{N} \sum_{k=1}^N (\omega_2(k) - \varphi^T(k-1) \theta)^2 \quad (7)$$

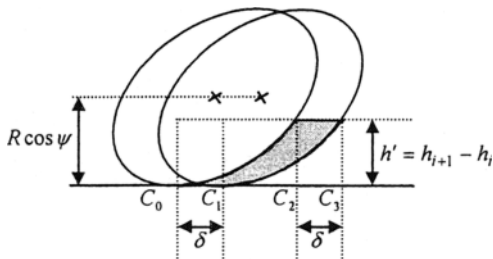
where  $\{i(k), \omega_2(k)\}$ ,  $k=1, 2, \dots, N$  are the input-output experimental data, and the regression vector is defined as  $\varphi^T(k-1) = [-\omega_2(k-1) \ -\omega_2(k-2) \ i(k-1)]$ . For more detailed and advanced adaptive identification/control schemes several literatures are available (Astrom and Wittenmark, 1995; Ljung, 1987; Hong and Bentsman, 1994; Hong, 1995, 1997; Van der Hof *et al*, 1995).

## 2.2 Output equation

In this subsection, an output equation that expresses the reclaiming rate is derived. The amount of raw material transported to the sinter



(a) Trajectories of the boom tip (top view)



(b) Advancement of the disk (side view)

Fig. 2 Geometries for obtaining an output equation.

plant is quantified as a function of the angular displacement and the angular velocity of the boom. Since a stacker uses the same rectilinear rail when it stacks raw materials in the raw yard, the raw material heaps are parallel to the rail. Figure 2(a) shows the trajectories of the bucket tip as viewed from the top in which the  $z_0$ -axis denotes the rectilinear rail. Arc  $AB$  shows the slew arc of the tip before the main body advances by  $d_1$ , and arc  $A'B'$  denotes the arc made after the advancement. Let  $\delta$  be the distance between the two intersection points made by the two arcs  $AB$  and  $A'B'$  and the boom. Then,  $\delta$  is a function of  $\theta_2$  given as

$$\delta(\theta_2) = (L + d_1 \cos \theta_2) - \sqrt{L^2 - d_1^2 \sin^2 \theta_2}. \quad (8)$$

Figure 2(b) shows a side view of the two rotating disks before and after the advancement of the main body by  $d_1$ . As the disk tilts by  $\phi$  and  $\psi$ , the shape of the disk becomes an ellipse. Let  $h' = h_{i+1} - h_i$  be the height of the shaded portion in Fig. 2 (b). This shaded portion indicates the volume of raw material that will be removed as the disk rotates. In general,  $h'$  may not be the same as  $R \cos \psi$ . Since the ellipse has been translated by  $\delta$ , the shaded area  $S$  is the same as the area of the rectangle whose base is  $C_2C_3$  and height is  $h'$ . Therefore,  $S$ , as a function of  $\theta_2$ , becomes

$$S(\theta_2) = \{(L + d_1 \cos \theta_2) - \sqrt{L^2 - d_1^2 \sin^2 \theta_2}\} \cdot h' \quad (9)$$

Finally, the reclaiming rate, as a function of  $\theta_2$  and  $\omega_2$ , is derived as

$$q(\theta_2, \omega_2) = \rho \cdot h' \cdot \omega_2 \cdot \{(L + d_1 \cos \theta_2) - \sqrt{L^2 - d_1^2 \sin^2 \theta_2}\} \quad (10)$$

where  $\rho$  is the density of the raw material. Let the linearization of Eq. (10) at an operating point, for example  $(\theta_2, \omega_2)_o = (45^\circ, 0.4^\circ/\text{sec})$ , be

$$q(\theta_2, \omega_2) = \left[ \frac{\partial q(\theta_2, \omega_2)}{\partial \theta_2} \quad \frac{\partial q(\theta_2, \omega_2)}{\partial \omega_2} \right]_o [\theta_2 \quad \omega_2]^T \quad (11)$$

where the subscript  $o$  denotes the linearization at the operating point.

### 2.3 Output delay

A large output measurement delay occurs due to the non-collocation of the end-effector (a bucket) and the output sensor (a load cell). The load cell is located 5 m from the rotational axis of the disk and below the conveyor belt. The size of delay varies depending on the type of the reclaimer. It ranges from 5 sec to 10 sec.

### 2.4 A nominal plant model

Note that Eq. (5) and Eq. (11) denote the system and the output equations of the plant, respectively. Let  $P(s)$  be the transfer function from  $i$  to  $q$ . Then, a nominal plant model, including the output delay, is defined as follows

$$\frac{q(s)}{i(s)} = P(s) \triangleq C_o(s) \cdot G_o(s) \cdot e^{-T_o s} \quad (12)$$

where  $C_o(s) = \left[ \frac{\partial q}{\partial \omega_2} + \frac{1}{s} \frac{\partial q}{\partial \theta_2} \right]_o$  from Eq. (11), is defined in Eq. (5), and  $T_o$  is a nominal value of the time delay. In this paper,  $T_o$  is 7.8 sec.

### 3. Control Design

In this section a feedback control of the reclaimer, for the purpose of uniform reclaiming, is investigated. To incorporate a large time delay in the output measurement, the time delay compensation scheme of a Smith predictor is investigated (Palmor, 1996). In addition, a robust stability criterion, in the presence of multiplicative uncertainty, is derived.

#### 3.1 Smith predictor

Figure 3 shows a block diagram of the Smith predictor and a plant with a multiplicative uncertainty.  $P(s)$  denotes the transfer function of the nominal plant model defined in Eq. (12), and  $\Delta(s)$  is the multiplicative uncertainty of the plant. This multiplicative uncertainty includes both the structured uncertainty contained in the parameter identification problem of Eq. (5) and the unstructured uncertainty neglected in deriving Eq. (4) and Eq. (10) such as the servo-valve spool dynamics, the pressure dynamics in the hydraulic pipes, and the uncertainties in the raw material piles.  $K(s)$  denotes the Smith predictor which consists of a primary controller  $K_o(s)$  and an inner feedback loop  $P_o(s) - P(s)$ , where  $P_o(s)$  denotes the plant model excluding the time delay, i. e.  $P_o(s) = C_o(s) G_o(s)$ . The inner feedback loop is called a predictor since the signal  $\hat{q}$  performs a prediction of the output by the delay units of time into the future. To enhance the control performance of the Smith predictor, two things are required. One is to know the exact time

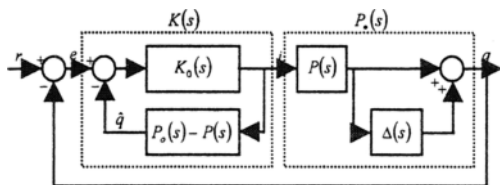


Fig. 3 Smith predictor configuration of the plant with a multiplicative uncertainty.

delay and the other is to have a good prediction model.

In the case of a reclaimer, the time delay can be measured once a specific reclaimer is chosen. However, as noted in the modeling stage of the plant equation in Sec. 2, the nominal plant model represented as Eq. (12) has a large uncertainty.

#### 3.2 A robust stability criterion

The transfer function of the plant with a multiplicative uncertainty is

$$P_*(s) = P(s) (1 + \Delta(s)) \tag{13}$$

where the subscript \* denotes the "true" plant. The transfer function of the Smith predictor is

$$K(s) = \frac{K_o(s)}{1 + K_o(s) (P_o(s) - P(s))} \tag{14}$$

The closed loop transfer function with  $\Delta(s) = 0$  is

$$T_c(s) = \frac{P(s) K(s)}{1 + P(s) K(s)} = \frac{K_o(s) P(s)}{1 + K_o(s) P_o(s)} \tag{15}$$

A robust stability criterion for the multiplicative uncertainty given by (Doyle *et al.*, 1992, p. 51) is

$$\|\Delta(j\omega) T_c(j\omega)\|_\infty < 1. \tag{16}$$

Note also that with  $P(j\omega) = P_o(j\omega) e^{-jT_o\omega}$  the following inequality holds

$$\begin{aligned} & \left\| \frac{\Delta(j\omega) K_o(j\omega) P(j\omega)}{1 + K_o(j\omega) P_o(j\omega)} \right\|_\infty \\ & \leq \left\| \frac{\Delta(j\omega) K_o(j\omega) P_o(j\omega)}{1 + K_o(j\omega) P_o(j\omega)} \right\|_\infty \end{aligned}$$

Therefore, criterion given in Eq. (16) for the Smith predictor becomes

$$\left\| \frac{\Delta(j\omega) K_o(j\omega) P_o(j\omega)}{1 + K_o(j\omega) P_o(j\omega)} \right\|_\infty < 1. \tag{17}$$

Finally, by utilizing Eq. (17), the following criterion for the primary controller  $K_o(s)$  is derived as follows

$$\left| \frac{K_o(j\omega) P_o(j\omega)}{1 + K_o(j\omega) P_o(j\omega)} \right| < \frac{1}{|\Delta(j\omega)|}, \text{ for } \forall \omega > 0. \tag{18}$$

Therefore, once  $|\Delta(j\omega)|$  of the plant is determined, the robustness of the Smith predictor can be assured by designing  $K_o(s)$  according to Eq. (18). In the next section, the uncertainty bound of

$\Delta(j\omega)$  for the reclaimer is derived.

#### 4. Uncertainty Quantification

In this section,  $1/|\Delta(j\omega)|$  of Eq. (18), by estimating the multiplicative model uncertainty  $\Delta(j\omega)$ , is quantified. Let the true transfer function of the plant be of the form

$$P_*(s) = P(s) (1 + \Delta(s)) = C_o(s) G_*(s) e^{-T_*s} \quad (19)$$

where  $G_*(s)$  denotes the true transfer function, in comparison with (5), from the input current to the angular velocity of the boom, and  $T_*$  is the true time delay. Note that Eq. (19) consists of the nominal  $C_o(s)$  from Eq. (12) instead of  $C_*(s)$ .

From Eq. (13), the following relationship holds for  $\omega > 0$

$$|\Delta(j\omega)| = \frac{|P(j\omega) - P_*(j\omega)|}{|P_o(j\omega)|} \quad (20)$$

Using Eq. (12) and Eq. (19), the numerator of Eq. (20) becomes

$$\frac{|P(j\omega) - P_*(j\omega)|}{|G_o(j\omega) e^{-j(T_o - T_*)\omega} - G_*(j\omega)|} \quad (21)$$

Since the time delay can be measured, assuming that  $T_o - T_* \cong 0$ , Eq. (20) becomes

$$|\Delta(j\omega)| \cong \frac{|G_o(j\omega) - G_*(j\omega)|}{|G_o(j\omega)|} \quad (22)$$

In this paper, Eq. (22) is estimated by experiments. The discrete time description of  $1/|\Delta(j\omega)|$  in Eq. (18), assuming that the sampling time is sufficiently small, is as follows (Astrom and Wittenmark, 1995, p. 123)

$$\frac{1}{|\Delta(e^{j\omega})|} = \frac{|G_o(e^{j\omega})|}{|G_o(e^{j\omega}) - G_*(e^{j\omega})|} \quad (23)$$

Since  $G_o(j\omega)$  is known, the question is how to estimate the denominator,  $|G_o(j\omega) - G_*(j\omega)|$ , of Eq. (23). By denoting  $G_*(j\omega) = G_o(j\omega) + G_\Delta(j\omega)$ , where  $G_\Delta(j\omega)$  is an additive uncertainty of  $G_o(j\omega)$ , the uncertainty quantification problem amounts to finding the magnitude bound of the additive uncertainty of the boom rotating dynamics. In this paper, the work of Goodwin *et al.* (1992) is used in estimating  $|G_\Delta(j\omega)|$ . The quantification procedure is summarized as fol-

lows.

##### ● Step 1

Use a parameter estimation method, which satisfies Eq. (7), to estimate the coefficients of the discrete nominal transfer function,  $G_o(z)$  of Eq. (6), with  $N$  point input and output data sequences,  $U = [u(1) \cdots u(N)]^T$  and  $Y = [y(1) \cdots y(N)]^T$ . In the case of a reclaimer,  $u$  is the input current  $i$ , and  $y$  is the boom rotational speed  $\omega_2$ .

##### ● Step 2

Use the parameter values estimated in Step 1 and convert the nominal plant model into the following form of a rational transfer function with a fixed denominator.

$$G_o(z) \cong \sum_{k=0}^{p-1} g(k) \lambda_k(z) = \Lambda(z) \hat{\theta}_g \quad (24)$$

For this function,  $p$  is a sufficiently large integer,  $g(k)$  are coefficients,  $\lambda_k(z)$  are appropriate basis functions for a stable linear time invariant system,  $\Lambda(z) = [\lambda_0(z) \lambda_1(z) \cdots \lambda_{p-1}(z)]$ , and  $\hat{\theta}_g = [g(0) \cdots g(p-1)]^T$ . For example, the basis functions of a FIR model are  $\lambda_k(z) = z^{-k}$ , and those of a Laguerre model are  $\lambda_k(z) = \frac{\sqrt{1-\xi^2}}{(z-\xi)^k}$ ,

where  $\xi$  is a positive real number. If Eq. (24) is expressed with a FIR model,  $g(k)$  is the impulse response of Eq. (6). If the Laguerre model is used,  $\xi$  is chosen to be the real part of the dominant pole of Eq. (6). Equation (24) is used instead of Eq. (6) because an expression of the plant transfer function, that is suitable for estimating the model uncertainty, is needed.

##### ● Step 3

Let  $\Pi(z) = [1 \ z^{-1} \cdots z^{-(m-1)}]$ , where  $m$  is an integer such that the impulse response of the true transfer function  $G_*(z)$  decays out sufficiently beyond the  $m$ -th step. Use the input data  $u(k)$  and the  $\Lambda(z)$  obtained in Step 2 to construct

$$\Phi^T = [\phi_1 \cdots \phi_N], \text{ where } \phi_k^T = \Lambda(z) u(k), \quad (25)$$

$$Q = (\Phi^T \Phi)^{-1} \Phi^T,$$

$$\Psi^T = [\psi_0 \cdots \psi_N], \text{ where } \psi_k^T = \Pi(z) u(k).$$

##### ● Step 4

Solve the maximization problem as given by

Goodwin *et al.* (1992). This maximization problem, for  $M$ ,  $\zeta$ , and  $\sigma_v$ , is defined by a log maximum likelihood function as

$$\underset{M, \zeta, \sigma_v}{\operatorname{argmax}} l(W; M, \zeta, \sigma_v) = \underset{M, \zeta, \sigma_v}{\operatorname{argmax}} \left( -\frac{1}{2} \ln \det \Sigma - \frac{1}{2} W^T \Sigma^{-1} W + \text{constant} \right) \quad (26)$$

where

$$W = H^T (Y - \Phi \hat{\theta}_g),$$

$H$  is a matrix which is composed of  $N$ - $p$  independent columns of  $I - \Phi Q$ ,

$$\Sigma = H^T \Psi C_\eta \Psi^T H + \sigma_v^2 H^T H,$$

$$C_\eta = \underset{0 \leq k \leq m-1}{\operatorname{diag}} [M \zeta^k].$$

● Step 5

For a specifically chosen frequency,  $\omega$ , calculate  $\Lambda(e^{-j\omega})$  and  $\Pi(e^{-j\omega})$ . Then, the expectation of the denominator of Eq. (23) is calculated as follows

$$E[|G_o(e^{-j\omega}) - G_*(e^{-j\omega})|^2] = (\Pi - \Lambda Q \Psi) C_\eta (\Pi - \Lambda Q \Psi)^* + \Lambda Q C_v Q^T \Lambda^* \quad (27)$$

where  $C_v = \sigma_v I_{N \times N}$ , and the superscript  $*$  denotes the complex conjugate transpose.

● Step 6

By repeating Step 5 at each  $\omega$ , the evaluation of Eq. (23) can be completed.

### 5. Experiments and Simulations

Experiments were carried out in the stockyard of Kwangyang Works of Pohang Steel and Iron Company, Ltd., in Korea.

● Step 1 of Section 5:

In identifying the unknown parameters in Eq. (6), 9000 point input-output data were sampled at a 15 Hz sampling frequency, i. e.  $N=9000$ . To fulfill the persistency of excitation condition of the input signal, three sinusoidal frequencies were added to the low speed slew motion of the boom. The three sinusoidal signals ranged from  $0.2^\circ/\text{sec}$  to  $0.6^\circ/\text{sec}$  or  $10^{-3.25} \sim 10^{-2.78}$  Hz. The identified transfer function of Eq. (6) is

$$G(z) = \frac{0.1429z^{-1}}{1 - 0.2592z^{-1} + 0.2166z^{-2}}. \quad (28)$$

Figure 4 compares the experimental data and

the time domain response of Eq. (28) with the same input sequence used in estimating Eq. (28). The validity of Eq. (28) has been improved by a number of experiments using various excitational signals. It is shown that the simulated results of Eq. (28) agree well with the experimental data.

● Step 2 and Step 3

In transforming the estimated transfer function of Eq. (6) into the form of Eq. (24), a fifth order Laguerre model ( $p=5$ ) with  $\xi=0.13$  is used, i.e.

$$G_o(z) = \sum_{k=0}^{\psi} g(k) \frac{\sqrt{1-0.13^2}}{(z-0.13)} \left[ \frac{1-0.13z}{z-0.13} \right]^k \quad (29)$$

where  $g(0)=0.0184$ ,  $g(1)=0.1454$ ,  $g(2)=-0.0120$ ,  $g(3)=-0.0287$ , and  $g(4)=0.0080$ . In calculating  $\Pi(z)$  in Step 3,  $m=30$  is used.

● Step 4:

In order to solve the maximization problem of Eq. (26), a genetic algorithm is used. The obtained values of  $M$ ,  $\zeta$ , and  $\sigma_v$  are

$$M=0.412, \zeta=0.793, \text{ and } \sigma_v=0.12. \quad (30)$$

The two exponentially decaying curves in Fig. 5 illustrate the additive uncertainty bound, given in Eq. (30), of the boom rotational dynamics.

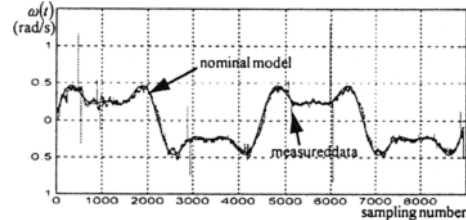


Fig. 4 Comparison between the experimental output and the response of (28).

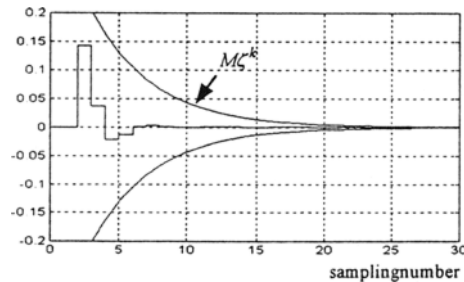


Fig. 5 An estimated additive uncertainty bound of  $G_o(jw)$  (experimental results) and the impulse response of (32).

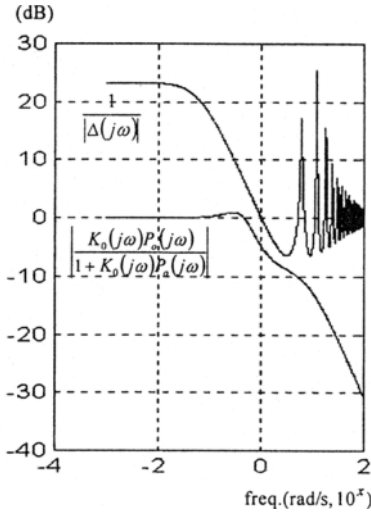


Fig. 6 Design criteria (18) obtained for a specific reclaimer (experimental results).

● Step 5 and Step 6:

The upper curve in Fig. 6 illustrates  $1/|\Delta(j\omega)|$  of (23) obtained through Step 5 and Step 6. The lower curve shows the magnitude plot of the right hand side of the robust Smith predictor criterion given in Eq. (18), where the primary controller  $K_o(s)$  is of the form

$$K_o(s) = 10.3 + 0.17 \frac{1}{s} + 20.2s \quad (31)$$

and the gains in Eq. (31) are determined by the loop shaping method in such a way that Eq. (18) is satisfied.

● Simulations:

To demonstrate the robust performance of the primary controller described by Eq. (31) with a time delay  $T_o = 7.8\text{sec}$ , the following additive uncertainty of the plant has been inserted.

$$G_\Delta(z) = \frac{-0.0334z^{-3} - 0.1726z^{-2} + 0.2284z^{-1} - 0.0224}{z^{-4} - 0.6567z^{-3} + 0.4550z^{-2} - 0.1212z^{-1} + 0.0293} \quad (32)$$

The time response between the two exponentially decaying curves in Fig. 5 depicts the impulse response of Eq. (32). Figure 7 compares the three step responses of a simple PID controller (best tuned), an ideal Smith predictor without any uncertainty in the model, and a robust Smith predictor that includes uncertainty described by Eq. (31). Compared with the simple PID, con-

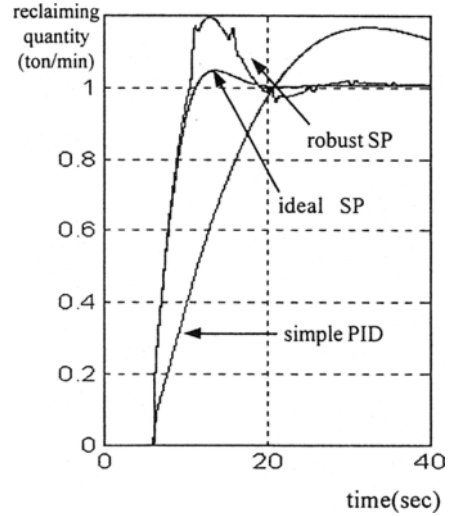


Fig. 7 Comparison of a simple PID, an ideal SP, and a robust SP (simulation results).

troller the robust Smith predictor performs well in both the rise and settling times.

### 6. Conclusions

In this paper, a nominal plant model and a robust Smith predictor control of a reclaimer were investigated. As the parameter values of the reclaimer were not available, the reclaiming process was modeled as a second order differential equation with unknown coefficients. The unknown coefficients were estimated by a recursive estimation method. Then a nominal plant model was defined using these estimated parameter values. In order to incorporate the large time delay in the output measurement, a Smith predictor was adopted as a feedback controller. A robust stability criterion of the Smith predictor was derived. To enhance the robustness of the closed loop system, the uncertainties that were neglected in the modeling and identification stage were estimated. Following the work of Goodwin *et al.* (1992), a design procedure that accounts for the uncertainties was illustrated.

### References

Astrom, K. J., and Wittenmark, B., 1995, *Computer Controlled Systems*, Prentice Hall,



Englewood Cliffs, NJ.

Choi, C., Lee, K., Shin, K., Hong, K. S., and Ahn, H., 1999, "Automatic Landing Method of a Reclaimer on the Stockpile," *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 29, Part C, No. 1 (May).

Doyle, J. C., Francis, B. A., and Tannenbaum, A. R., 1992, *Feedback Control Theory*, Maxwell Macmillan, New York.

Goodwin, G. C., Gevers, M., and Ninness, B., 1992, "Quantifying the Error in Estimated Transfer Functions with Application to Model Order Selection," *IEEE Transactions on Automatic Control*, Vol. 37, No. 7, pp. 913~927.

Han, M. C., Hong, K. S., Yoon, J., and Lee, S., 1997, "Decentralized Robust Control for Interconnected Nonlinear Systems," *KSME International Journal*, Vol. 11, No. 1, pp. 1~9.

Hong, K. S. and Bentsman, J., 1994, "Direct Adaptive Control of Parabolic Systems: Algorithm Synthesis, and Convergence and Stability Analysis," *IEEE Transactions on Automatic Control*, Vol. 39, No. 10, pp. 2018~2033.

Hong, K. S., 1995, "New Convergence Analysis in Adaptive Control: Convergence Analysis without Barbalat's Lemma," *KSME International Journal*, Vol. 9, No. 2, pp. 138~146.

Hong, K. S., 1997, "Asymptotic Behavior Analysis of a Coupled Time-Varying System: Application to Adaptive Systems," *IEEE*

*Transactions on Automatic Control*, Vol. 42, No. 12, pp. 1693~1697.

Hong, K. S., Kim, Y. M., and Choi, C., 1997, "Inverse Kinematics of a Serial Manipulator: Redundancy and a Closed-form Solution by Exploiting Geometric Constraints," *KSME International Journal*, Vol. 11, No. 6, pp. 629~638.

Ljung, L., 1987, *System Identification: Theory for the User*, Prentice Hall, Englewood Cliffs, NJ.

Palmor, Z. J., 1996, "Time-Delay Compensation- Smith Predictor and its Modifications," *The Control Handbook*, Vol. 1, CRC press, pp. 224~229.

Phillips, C. L., and Nagle, H. T., 1995, *Digital Control System Analysis and Design*, Prentice Hall, Englewood Cliffs, NJ.

Van der Hof, P. M. J., Heuberger, P. S. C., and Bokor, J., 1995, "System Identification with Generalized Orthonormal Basis Functions," *Automatica*, Vol. 31, No. 12, pp. 1821~1834.

Watton, J., 1989, *Fluid Power Systems*, Prentice-Hall, Englewood Cliffs, NJ.

Wu, J. W. and Hong, K. S., 1994, "Delay-Independent Exponential Stability Criterion for Time-Varying Discrete Delay Systems," *IEEE Transactions on Automatic Control*, Vol. 39, No. 4, pp. 811~814.